AD-A083 963 BROWN UNIV PROVIDENCE R I DIV OF ENGINEERING F/G 12/1
CONTROL OF NONLINEAR SYSTEMS.(U)
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LEVEL III

Final Technical Report

for

CONTROL OF NONLINEAR SYSTEMS

to

Directorate of Mathematical and Information Sciences
Department of the Air Force
Air Force Office of Scientific Research (AFSC)
AFOSR/NM, Building 410
Bolling Air Force Base
Washington, DC 20332

Grant: AFOSR-75-2793

Period Covered: January 1, 1975-December 31, 1979

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Division of Engineering
Providence, Rhode Island 02912

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February 26, 1980

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR-TR-80-0252	2. GOVT ACCESSION NO.: AD-A083 963
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): CONTROL OF NONLINEAR SYSTEMS	5. TYPE OF REPORT & PERIOD COVERED: FINAL
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Allan E. Pearson	8. CONTRACT OR GRANT NUMBER(s): AFOSR-75-2793
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Brown University, Division of Engineering, Providence, RI 02912	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F 2304/A1
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research/NM, Bolling AFB, Washington, D.C. 20332	12. REPORT DATE: Feb~~ruary 26~~, 1980
	13. NUMBER OF PAGES: 21
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Control Systems, Missile Intercept Guidance and Control, Feedback Stabilization Time Varying Systems, Nonlinear Systems, System Identification, Signal Estimation.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research ~~carried out under AFOSR 75-2793~~ falls under six main cagegories: ~~(A)~~ feedback stabilization methods for linear systems, including time varying, discrete,c ontinuous, differential-delay and linear systems subject to average power constraints. ~~(B)~~ Controllability for a class of nonlinear systems. ~~(C)~~ Minimum energy regulators for commutative bilinear systems. ~~(D)~~ Control laws for certain aerospace applications. ~~(E)~~ Least squares parameter identification for linear and nonlinear systems. ~~(F)~~ Signal reconstruction. These results are summarized ~~below~~ for each category.

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I. INTRODUCTION

This progress report covers the five year period preceding December 31, 1979 and is the final (and fifth) such progress report filed under Grant AFOSR-75-2793. The personnel listed below received at least partial support from the grant during some portion of the tenure of the grant. The research completed during this period is discussed in Section III.

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III. COMPLETED RESEARCH

The research carried out under AFOSR-75-2793 falls under six main categories: (A) Feedback stabilization methods for linear systems [1,2,5,6,8,9,14]†, including time varying, discrete, continuous, differential-delay and linear systems subject to average power constraints. (B) Controllability for a class of nonlinear systems [3,10]. (C) Minimum energy regulators for commutative bilinear systems [3,10]. (D) Control laws for certain aerospace applications [7,11]. (E) Least

† Brackected numbers refer to similarly numbered articles listed chronologically in Section V.

squares parameter identification for linear and nonlinear systems [4,13,15]. (F) Signal reconstruction [16]. These results are summarized below for each category.

A. <u>Feedback Stabilization Methods for Linear Systems</u>

Consider the linear differential system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \ , \quad x(t_o) = x_o \qquad (1)$$
$$y(t) = C(t)x(t)$$

and the positive definite quadratic cost function

$$J(u) = \int_{t_o}^{t_1} [y'(\tau)\, Q(\tau)\, y(\tau) + u'(\tau)R(\tau)u(\tau)]d\tau \ . \qquad (2)$$

A "finite horizon time" optimal control results if (2) is minimized over u, with t_o and t_1 selected according to

$$t_o = t \ , \quad t_1 = t + T \qquad (3)$$

for some fixed $T > 0$, and the state vector is subject to the moving terminal constraint

$$x(t + T) = 0 \ . \qquad (4)$$

The initial value for this solution, i.e. $u^*(t,x(t))$, has been shown in [8] to result in a stabilizing feedback control law for (1). This control law is given as follows:

$$u(t,x(t)) = -R^{-1}(t)B'(t)P^{-1}(t,t + T)x(t) \qquad (5)$$

where $P(t,t + T)$ is obtained by integrating the following matrix Riccati equation backward in time from $\tau = \sigma = t + T$ to $\tau = t$:

$$\frac{-\partial P(\tau,\sigma)}{\partial \tau} = -A(\tau)P(\tau,\sigma)-P(\tau,\sigma)A'(\tau)-P(\tau,\sigma)C'(\tau)\, Q(\tau)\, C(\tau)P(\tau,\sigma) + B(\tau)R^{-1}(\tau)B'(\tau) \ ,$$
$$\tau \leq \sigma \ , \quad P(\sigma,\sigma) = 0 \ . \qquad (6)$$

Under suitable controllability and observability assumptions about the pairs $[A(t),B(t)]$ and $[A(t),C(t)]$, the control law (5) renders the system (1) asymptotically stable even though it stems from the optimal control for minimizing (2) over a finite time interval of length T subject to the receding horizon constraint (4). This is not surprising if one specializes to the time invariant case and chooses $Q = 0$ in (2). In this case (as shown by Theorem 3.1 in [8]), the control (5) reduces to Kleinman's fixed gain control law (Kleinman, D. L., "An Easy Way to Stabilize a Linear Constant System," <u>IEEE Trans. on Auto. Contr.</u>, Vol. AC-15, p. 692, 1970):

$$u(t) = -R^{-1}B'W^{-1}(T)x(t) \tag{7}$$

where $W^{-1}(T)$ is the inverse of the controllability Gramian:

$$W(T) = \int_0^T e^{-At}BR^{-1}B'e^{-A't}dt \ . \tag{8}$$

Notwithstanding this property, it is important to point out that the control (5) requires the integration of a Riccati equation over a <u>finite</u> time interval in contrast with the asymptotic optimal control for the standard linear regulator problem which requires an infinite time integration interval in the case of time varying systems. Variants of this control law have also been used to derive new feedback stabilization methods for a class of linear differential difference systems [6] described by

$$\dot{x}(t) = Ax(t) + A_h x(t - h) + Bu(t) \ , \tag{9}$$

and time varying discrete systems [9] described by

$$x_{i+1} = \Phi_i x_i + B_i u_i \ , \tag{10}$$

as well as time invariant discrete systems [1].

Recently, a new finite horizon time control law was developed [14] motivated by an investigation of the tolerance to nonlinearities property of the control (5). This control stems from the optimal control for minimizing the double integral

quadratic cost

$$J(u) = \int_t^{t+T} \int_t^{\tau} u'(s)R(s)u(s)dsd\tau \tag{11}$$

subject to the double integral finite horizon time constraint

$$\int_t^{t+T}\int_t^{\tau}\Phi(t,s)B(s)u(s)dsd\tau = -x(t) \ , \tag{12}$$

where $\Phi(t,s)$ is the state transition matrix for $A(t)$ in (1). This new control law takes the form

$$u(t,x(t)) = -R^{-1}(t)B'(t)\tilde{P}^{-1}(t,t+T)x(t) \tag{13}$$

where $\tilde{P}^{-1}$ is the inverse of the pure integration of the matrix solution to (6), i.e.

$$\tilde{P}(t,t+T) = \int_t^{t+T}P(t,\tau)d\tau \ , \quad T > 0 \tag{14}$$

where $P(t,\tau)$ satisfies (6).

Although not particularly obvious, the new control (13) asymptotically stabilizes the system (1) (under appropriate observability and controllability conditions for (1)). (See Theorem 1 in [14].) More importantly, the control law (13) exhibits a significantly greater tolerance for nonlinearities in the loop without destroying stability in comparison with (5). For example, in the case of single input time invariant systems, the control (13) reduces to

$$u = -R^{-1}\tilde{P}^{-1}(T)x \tag{15}$$

where $\tilde{P}(T) = \int_0^T P(t)dt$ and $P(t)$ satisfies

$$\dot{P} = -AP-PA'-PC'QCP + BR^{-1}B' \ , \quad P(o) = 0 \ . \tag{16}$$

The control (15) tolerates nonlinearities in the first and third quadrant with slopes bounded below by $T/2$. This means that with a given nonlinearity in the first and third quadrant, it is always possible to find a suitably small positive T in (15) so that the resulting feedback control system is asymptotically stable in spite of the nonlinearity. By contrast, the control (5) tolerates nonlinearities to the extent that their lower bound slope is unity in the first and third quadrant.

Equally important, it has been shown that the feedback system employing (15) possesses infinite gain margin for any T satisfying $0 < T \leq 2$ and a phase margin at least as great as

$$\theta_m = \tan^{-1}\sqrt{\frac{4}{T^2} - 1} .$$

(Theorem 3.1) in [14].) Hence, the phase margin can be made to approach $+90°$ as $T \to 0$. By contrast, the feedback control law for the standard steady state optimal regulator has been shown to be insensitive to nonlinearies which are slope restricted in the first and third quadrants with a lower bound gain of one-half, and to possess a phase margin with bound $+60°$ (Safonov, M. G. and Athans, M., "Gain and Phase Margin for Multiloop LQG Regulators," IEEE Trans. on Auto. Contr., Vol. AC-22, pp. 173-179, 1977).

Interest in the above type of feedback control stems from early work on minimum energy controllers by the PI (Pearson, A. E., "Synthesis of Minimum Energy Controllers Subject to an Average Power Constraint," IEEE Trans. on Appl. and Ind., No. 66, pp. 70-75, 1963). Specifically, minimizing the control energy

$$J(u) = \int_t^{t+T} u'(\tau)\, Ru\,(\tau) d\tau$$

subject to the terminal constraint (4) in the time invariant case leads to the open loop optimal control

$$u(\tau, x(t)) = -R^{-1}B'e^{-A'(\tau-t)}W^{-1}(T - t)x(t) , \quad t \leq \tau \leq t + T \qquad (17)$$

which reduces to (7) at the initial time $\tau = t$. However, if the open loop control (17) is now subjected to the "average power" constraint

$$\frac{1}{T}\int_t^{t+T} u'(\tau) R\, u\,(\tau) d\tau = 1 \qquad (18)$$

the result is a constraint on the positive parameter T in that (17)-(18) yields

$$\frac{1}{T - t}\, x'(t)W^{-1}(T - t)x(t) = 1 . \qquad (19)$$

Combining (17) with (19) and choosing $\tau = t$ in (17) results in the following

<u>implicitly defined</u> nonlinear control law

$$u(x) = -R^{-1}B'W^{-1}(T(x))x \quad , \quad x \neq 0 \tag{20}$$

where $T(x)$ is the unique positive solution to the transcendental equation

$$\frac{1}{T}x'W^{-1}(T)x = 1 \; . \tag{21}$$

Although nonlinear and defined implicitly, the control (20)-(21) has been shown in [2] to be asymptotically stabilizing for the fixed version of (1). A design procedure has also been given in Section III of [2] which facilitates construction of an <u>explicitly defined closed loop</u> control approximating (20)-(21) to any desired degree of accuracy. The importance of this control law is that it provides a judicious balance between fast response time and a modest degree of control effort over a large region in the state space because it tends to allocate the control effort more evenly over the response time than a conventional linear feedback control. This is demonstrated by simulation results in [2] and is justified intuitively by the average power constraint (18) which is also satisfied by "bang-bang" controllers.

New lower bounds for the solution to the algebraic matrix Riccati equation were obtained in [5]. (See also Fahmy, M. M. and Hanafy, A. A. R., "Comments on 'A Note on the Algebraic Matrix Riccati Equation'", <u>IEEE Trans. on Auto. Contr</u>. Vol. AC-24, p. 143, 1979, and the Author's Reply on the same page.) Recently, sharper bounds have been obtained (Yasuda, K. and Hirai, K., "Upper and Lower Bounds onthe Solution of the Algebraic Riccati Equaiton," <u>IEEE Trans. on Auto. Contr</u>., Vol. AC-24, pp. 483-487, 1979).

B. <u>Controllability for a Class of Nonlinear Systems</u>

Sufficient conditions for the controllability of the class of nonlinear

systems described by

$$\dot{x}(t) = A(t,x(t),u(t))x(t) + B(t)u(t) + f(t,x(t),u(t)) \ , \ t_o \leq t \leq t_1$$

have been obtained by Wei [3] during this period. These conditions involve the nonsingularness of the controllability Gramian associated with the parametrized matrix pair $\{A(t,\zeta(t),v(t)),B(t)\}$, where $\zeta(t)$ and $v(t)$ are regarded as elements (parameters) in a product space, $C_{nm}[t_o,t_1]$, of vector valued continuous function pairs, $(\zeta(t),v(t))$, on the time interval $t_o \leq t \leq t_1$. Using the Schauder's fixed point theorem in $C_{nm}[t_o,t_1]$, sufficient conditions for both local and global controllability are derived involving the boundedness and continuity of the quantities $(A(t,x,u),B(t),f(t,x,u))$ and their partial derivatives, in addition to the nonsingularness of the aforementioned controllability Gramain. These results remove some assumptions previously needed in earlier publications on this problem and, generally, extend these earlier results to a broader class of nonlinear systems.

Again using fixed point arguments, sharper results were obtained in [11] for the global controllability of the more restricted class of nonlinear systems represented by the bilinear state equations

$$\dot{x}(t) = \left[A(t) + \sum_{i=1}^{m} B_i(t)u_i(t)\right]x(t) + C(t)u(t) \ .$$

In particular, certain boundedness assumptions were removed relative to previous results: (Rink, R. E. and Mohler, R. R., "Completely Controllable Bilinear Systems," SIAM J. on Contr. 6, pp. 477-486, 1968; Klamka, J. "Controllability of Nonlinear Systems With Delay in Control," IEEE Trans. on Auto Contr., Vol. AC-20, pp. 702-704, 1975, and [3].)

C. Minimum Energy Regulators for Commutative Bilinear Systems

Consider the class of commutative homogeneous bilinear systems

$$\dot{x} = \left[A + \sum_{i=1}^{m} B_i u_i\right] x \tag{22}$$

where each pair of matrices in the constant set $\{A, B_1, \ldots B_m\}$ commute with one another, together with the quadratic cost function

$$J(u) = x'(t_1)Fx(t_1) + \int_{t_0}^{t_1} u'(t)Ru(t)dt \ . \tag{23}$$

An example of a class of physical systems which fits within the framework of (22) is the "pursuit-evasion" problem in which the equations of motion are

$$\dot{x} = -v_T \sin\beta + u_p y$$
$$\dot{y} = v_T \cos\beta - u_p x - v_p$$
$$\dot{\beta} = u_T - u_P \tag{24}$$

Here, (v_p, v_T) and (u_p, u_T) are the line speeds and turning rates respectively for a pursuer, P , and a target, T , with β the relative heading between a pursuer and a target in the plane. As shown in [12], (24) can be placed in the form of (22) with $m = 1$, $u = u_P$ being the primary control variable, and assuming that the pursuer possesses both thrust modulation and thrust vectoring capabilities, i.e. $AB = BA$ holds for all values of the target parameters (u_T, v_T).

Concerning the general commutative bilinear system (22), it is shown in [12] that the optimal control which minimizes (23) without terminal constraints is in the form of a constant vector which satisfies a certain nonlinear algebraic equation. Furthermore, for a single input commutative bilinear system $(m = 1)$, it is shown in [12] that this optimal control is unique if (as a sufficient condition) the matrix $B_1'Q + B_1'QB_1$ is nonnegative definite. Also, sufficient conditions have been obtained in the multi-input case which involves the non-

negative definiteness for all $v \varepsilon R^n$ of the mxm matrix $Z(v)$ defined by

$$Z_{ij} = v'(B_j'B_i'Q + B_i'QB_j)v \ , \quad i,j = 1 \ , \ .. \ m \ .$$

The implication of these results for the regulator problem associated with a commutative bilinear system is that the optimal control can be computed by well-known iterative methods in finite dimensional (R^m) spaces, and that this control vector is unique if certain additional conditions involving the system matrices are upheld.

Concerning the same class of regulator problems for commutative bilinear systems, but with a fixed terminal state constraint, i.e., $x(T) = x_1$ = a given terminal vector, it has also been shown in [12] that if x_1 belongs to the reachable set, then there exists a constant optimal control which does the job, and that this optimal control vector satisfies a certain nonlinear algebraic equation which depends on the given boundary conditions: $x(t_o) = x_o$ and $x(T) = x_1$. In the terminal constraint problem such optimal controls are not generally unique and a simple example is given in [12] to illustrate this fact.

D. Control Laws for Certain Aerospace Applications

The implication of the results in [12] for the pursuit-evasion problem described by (24) is the following: (i) With the assumption that the pursuer's turning rate $u_p(t)$ vanishes only at points of measure zero, i.e. a switching type function, the pursuer's line speed can be modeled by $v_p(t) = \gamma(t)u_p(t)$ without any loss of generality, where $\gamma(t)$ is some scaling factor. Under these circumstances the bilinear system corresponding to (24) (see Eq.(5) in [12] is commutative and, hence, the optimal control for the zero missed distance case, i.e. the terminal constraint situation

$$x(T) = y(T) = 0 \quad \text{for some} \quad T > t_o \ , \qquad (25)$$

is simply a constant vector determined by the boundary conditions (Theorem 2

in [12]. (ii) As summarized by Propositions 1 and 2 in [11], there does indeed exist a triple $(\gamma, \beta(T), T)$ for every set of initial data $(x(t_o), y(t_o), \beta(t_o))$ with γ a constant parameter, such that (25) is satisfied for some $T > t_o$. Moreover, it is possible to solve the nonlinear algebraic equations explicitly (though nonuniquely) for $(\gamma, \beta(T), T)$.

Using the above results together with an on-line least squares identification technique for estimating the target parameters (u_T, v_T) and the initial heading $\beta(t_o)$, a step-by-step estimation and control sequence has been devised in [11] which provides an ad hoc (closed loop) feedback control law for the above described class of pursuit-evasion problems.

A singular perturbation problem has also been considered in [11] relating to the practical situation in which the missile turn rate is furnished by a motor with actuator dynamics. First order dynamics were assumed for the analysis and simulation studies, but the results actually apply to higher order actuator dynamics as well. An interesting feature of these results is that a closed-form solution can be obtained for the higher order singularly perturbed system of this paper in contrast with the approximate solutions for general nonlinear systems.

E. Least Squares Parameter Identification for Linear and Nonlinear Systems

A deterministic least squares identification of the coefficient matrices in the differential operator model

$$P(D)y(t) = Q(D)u(t) \ , \quad D = \frac{d}{dt}$$

where

$$P(D) = D^n + \sum_{i=o}^{n-1} P_{n-i} D^i \ , \quad Q(D) = \sum_{i=o}^{n-1} Q_{n-i} D^i \ ,$$

has been developed in [4] which differs from more traditional uses of least squares theory in the following respects: (i) input-output data $[u(t), y(t)]$

is assumed to be given on a finite time interval, $0 \leq t \leq t_1$, of arbitrarily short (but non-zero) duration, (ii) unknown disturbance inputs and measurement noises on $0 \leq t \leq t_1$, are modeled implicitly in the above model by arbitrary solutions to a homogeneous linear differential equation of assumed order, but with no assumptions about the characteristic modes of this equation, (iii) no attempt is made to estimate either the initial state of the system or the initial conditions giving rise to the disturbance inputs on $0 \leq t \leq t_1$.

One advantage of this approach, which might be termed parameter identification without initial state estimation, is that the potential exists for obtaining very accurate estimates of the system parameters, based on input-output data observed over a relatively short time interval, even for very small signal-to-noise ratios, e.g. -20db. or less. The main reason for this lies in the technique developed in [4] for circumventing the need to estimate the unknown initial conditions, which reduces this aspect of the computational burden associated with other approaches. Another reason is that the disturbances are modeled deterministically as uncontrollable modes, and the frequencies associated with these modes on $0 \leq t \leq t_1$ are identified along with the system parameters.

Theoretical conditions for the uniqueness of solutions to the above least squares estimation problem have also been obtained in [4]. These conditions involve the linear independence of the given input-output data, together with a certain number of their derivatives on $[0,t_1]$. Simulation results are reported in [4] which illustrate that highly accurate estimates for the parameters of a fourth order system can be obtained on a time interval comparable to the time constants in the system even in the presence of very large disturbance signals.

Important extensions in the above formulation have been obtained which enlarge the class of systems and provide for computational advantages in a variety of situations [13,15]. These extensions arise principally by viewing the identification problem in terms of finding a parameter vector θ which satis-

fies a differential operator equation of the form

$$P(D)v(t) + Q(D)V(t)f(\theta) = 0 \ , \quad 0 \leq t \leq t_1$$

where $(P(D),Q(D))$ are given polynomial matrices in the differential operator $D = \frac{d}{dt}$, $(v(t),V(t))$ are vector and matrix valued functions of the given input-output data, $f(\theta)$ is a given vector valued function (possibly nonlinear) of the parameter vector θ , and the observation time interval, $0 \leq t \leq t_1$, is again of arbitrarily short duration. More generally, the new formulation applies to any parametrized dynamical system whose differential operator equation can be arranged into the form

$$R(D)g(t,\omega) = S(D,\omega)d(t) \tag{26}$$

where $g(t,\omega)$ is a given vector valued function of the input-output data on $[0,t_1]$ and system parameter vector ω , $(R(D),S(D,\omega))$ are polynomial matrices in D with the coefficients of S possibly depending on the unknown system parameters but independent of time, and $d(t)$ is an unknown disturbance modeled on $[0,t_1]$ by the solution to a homogeneous linear differential operator equation of the form

$$T(D,\delta)d(t) = \sum_{i=o}^{r} \delta_i D^{r-i} d(t) = 0 \ . \tag{27}$$

$$\delta_o \triangleq 1 \ .$$

Operating on both sides of (26) with $T(D,\delta)$ and selecting a square nonsingular polynomial matrix $F(D)$ of sufficiently high order so that $F^{-1}(s)R(s)s^r$ is a proper transfer function matrix, the equation error function $z(t,\theta)$, $\theta = (\delta,\omega)$, is defined implicitly through the <u>linear</u> differential operator equation

$$F(D)z(t,\theta) = [R(D)D^r, \ .. \ R(D)] \begin{bmatrix} \delta_o(t,\omega) \\ \vdots \\ \delta_o g(t,\omega) \end{bmatrix} \ , \quad 0 \leq t \leq t_1 \tag{28}$$

Projecting the solution $z(t,\theta)$ to (28) down into a subspace devoid of all initial condition responses to (28) (via the same annihilating filter $\underline{H}$ as introduced in [4] to zero the initial condition response of a linear system

on a fixed finite time interval, i.e., Eq. (22) in [4] or Eq. (A in [1])

$$\tilde{z}(t,\theta) = \underline{H}(z(t,\theta))$$

and forming the inner product norm of $\tilde{z}(t)$ leads to the functional for least squares minimization:

$$J_1(\theta) = \langle\tilde{z}(t), \tilde{z}(t,\delta)\rangle = \int_0^{t_1} \tilde{z}'(t,\theta)\tilde{z}(t,\theta)dt \ . \tag{29}$$

As shown in [13] this functional possesses the interesting property of reducing to an explicitly defined function of the parameter vector $\theta = (\delta,\omega)$ in the case of systems which are "separable in the parameters". Basically, these are system models for which the $g(t,\omega)$ vector in (26) can be written as

$$g(t,\omega) = U(t)h(\omega)$$

for $U(t)$ a matrix valued function of the data on $[0,t_1]$ and $h(\omega)$ is a vector valued function of the system parameters ω . This is the case for all linear differential systems, a class of bilinear system models, as well as such special system models as the Duffing, Hammerstein and Van der Pol equations. Examples of nonseparable models are differential-delay systems with unknown time lag and Mathieu's equation with an unknown frequency for the time varying sinusoidal coefficient.

The computational advantages of separable models for this formulation are discussed in [13], although developments along these lines are still continuing. In addition, special results have been obtained to take advantage of "partially decoupled" parametrized MIMO systems by breaking the parameter estimation problem down into a finite sequence of lower dimensional function minimizations [15]. The main advantage of this is that it is possible to accomodate a higher order disturbances model for a given time interval $[0,t_1]$, while still obtaining satisfactory accuracy in the system parameter estimates, in relation to that which could be accomodated for a nondecoupled formulation. Alternatively, it is possible to obtain satisfactory accuracy in the system parameter estimates

for a shorter time interval $[0,t_1]$ than is possible if one fails to take advantage of the partial decoupling. These advantages were realized in the results reported in [15].

F. Signal Reconstruction

An application of the formulation for least squares identification described in the previous section has resulted in a filter for the classic problem of reconstructing the signal $s(t)$ based on the observation

$$y(t) = s(t) + v(t) \ , \quad 0 \leq t \leq t_1 \tag{30}$$

where the models for the "signal" $s(t)$ and the "noise" $v(t)$ are the differential operator equations:

$$s(t) : P(D)s(t) = Q(D)u(t) \tag{31}$$

$$v(t) : R(D,\theta)v(t) = \sum_{i=o}^{r} \theta_i D^{r-i} v(t) = 0 \tag{32}$$

$$\theta_o \triangleq 1 \ .$$

In the above, the differential operators P and Q are assumed given along with the forcing function $u(t)$ on $[0,t_1]$, and r is a preselected integer for the disturbance signal similar to the model (27).

Since the data is given on a finite time interval $[0,t_1]$, a natural choice for the signal model is to assume $u(t) = 0$ and take

$$P(D) = D \prod_{k=1}^{m} (D^2 + k^2\omega_o^{\ 2}) \ , \quad \omega_o = \frac{2\pi}{t_1} \ . \tag{33}$$

This is the finite Fourier series representation for the signal $s(t)$ and the problem is to estimate the Fourier coefficients in the presence of the modal type noise modeled by (32). Other signal models are obviously possible, due consideration being given to complex conjugate mode location so the P and Q polynomials are real. The total number of degrees of freedom consists of

the arbitrary initial conditions for (31) and (32) and the parameters $\theta = (\theta_1 . \, . \, . \, \theta_r)$ in (32). Such degrees of freedom can be regarded as comprising $2r + n$ independent random variables for generating a finite time stochastic process $y(t)$ where n is the degree of $P(D)$. However, the density functions for these random variables are not presumed to be known, nor need they be estimated.

The filter which has been devised in [16] for the above problem first estimates the modes in the disturbance signal $v(t)$, via an estimate of θ, then utilizes this estimate to reconstruct the signal $s(t)$ by solving for the unknown initial conditions in (30)-(32). This estimate is theoretically exact, barring computational errors, under the conditions: (i) the modal representation for the signal $s(t)$ and noise $v(t)$ are correct, and (ii) the modes present in $s(t)$ and $v(t)$ are disjoint over $[0,t_1]$. We shall not describe the details of this filter here, since it is basically an application of the theory described in the previous section. However, it is important to recapitulate that the filter yields a least squares estimate of $s(t)$ when modeling errors in (31) and (32) occur, and that the filter mitigates the effect of such errors provided the spurious signals possess an energy content which is small compared to the correctly modeled signals (rougly less than ten per cent).

It has been demonstrated through simulation that the filter yields excellent recovery of the signal $s(t)$, even for extremely small signal-to-noise ratios, e.g. $SNR = 20 \log_{10} ||s||/||v||$ on the order of -40 db or less, and in the presence of a small amount of "white" measurement noise, provided the modes of $s(t)$ and $v(t)$ are separated by a distance in the complex plane of at least $\omega_o = 2\pi/t_1$. This is also justified on an intuitive basis since ω_o is the fundamental frequency associated with the finite time interval $[0,t_1]$.

IV. CONTINUING RESEARCH

Work in progress which was supported by AFOSR-75-2793 and will appear in the near future includes the following papers:

Kwon, W. H. and Pearson, A. E., "Feedback Stabilization of Linear Systems With Delayed Control", to appear in IEEE Trans. on Auto. Contr., Vol. AC-25, April 1980.

Pearson, A. E., "Equation Error Identification With Modal Disturbances Suppressed," presented at a workshop on "Numerical Techniques for Stochastic Systems," University of Milan, Italy, September, 1979. Proceedings to be published in 1980.

The first of the above papers shows how the "finite horizon time" feedback stabilization technique discussed in Section III-A can be extended to derive stabilizing control laws for the linear differential system with delayed controls:

$$\dot{x} = Ax(t) + B_o u(t) + B_1 u(t - h) .$$

The second of the above papers shows how the formulation of Section III-E for least squares parameter identification can be further developed to result in a functional $J(\omega)$ depending solely on the system parameters, i.e. with the disturbance parameter vector δ in (27) effectively eliminated. This has important implications for the recursive identification of nonlinear systems with random modal disturbances currently under development.

V. PUBLICATIONS UNDER AFOSR-75-2793

A. Journal Articles

[1] Kwon, W. H. and Pearson, A. E., "On the Stabilization of a Discrete Constant Linear System," IEEE Trans. on Auto. Contr., Vol. AC-20, pp. 800-801, December 1975.

[2] Pearson, A. E. and Kwon, W. H., "A Minimum Energy Feedback Regulator for Linear Systems Subject to an Average Power Constraint," IEEE Trans. on Auto. Contr., Vol. AC-21, pp. 757-761, October 1976.

[3] Wei, K. C., "A Class of Controllable Nonlinear Systems," IEEE Trans. on Auto. Contr., Vol. AC-21, pp. 787-789, October 1976.

[4] Pearson, A. E., "Finite Time Interval Linear System Identification Without Initial State Estimation," Automatica, Vol. 12, pp. 577-587, November 1976.

[5] Kwon, W. H. and Pearson, A. E.,"A Note on the Algebraic Matrix Riccati Equation," IEEE Trans. on Auto. Contr., Vol. AC-22, pp. 143-144, February 1977.

[6] Kwon, W. H. and Pearson, A. E., "A Note on Feedback Stabilization of a Differential Difference System," IEEE Trans. on Auto. Contr., Vol. AC-22, pp. 468-470, June 1977.

[7] Pearson, A. E., Wei, K. C. and Koopersmith, R. M., "Terminal Control of a Gliding Parachute in a Non-uniform Wind," AIAA Journal, Vol. 15, pp. 916-922, July 1977.

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[9] Kwon, W. H. and Pearson, A. E., "On Feedback Stabilization of Time Varying Discrete Linear Systems," IEEE Trans. on Auto. Contr., Vol. AC-23, pp. 479-481, June 1978.

[10] Wei, K. C. and Pearson, A. E., "Global Controllability for a Class of Bilinear Systems," IEEE Trans. on Auto. Contr., Vol. AC-23, pp. 486-488, June 1978.

[11] Wei, K. C. and Pearson, A. E., "Control Law for an Intercept System," AIAA J. on Guid. and Contr., Vol. 1, No. 5, pp. 298-304, October, 1978.

[12] Wei, K. C. and Pearson, A. E., "On Minimum Energy Control of Commutative Bilinear Systems," IEEE Trans. on Auto. Contr., Vol. AC-23, pp. 1020-1023, Dec. 1978.

[13] Pearson, A. E., "Nonlinear System Identification with Limited Time Data," Automatica, Vol. 15, pp. 73-84, January 1979.

[14] Kwon, W. H. and Pearson, A. E., "A Double Integral Quadratic Cost and Tolerance of Feedback Nonlinearities," IEEE Trans. on Auto. Contr., Vol. AC-24, pp. 445-449, June 1979.

[15] Pearson, A. E. and Chin, Y. K., "Identification of MIMO Systems with Partially Decoupled Parameters," IEEE Trans. on Auto. Contr. Vol. AC-24, Aug. 1979.

[16] Pearson, A. E. and Mocenigo, J. M., "A Filter for Separating Finite Time Modal Signals," IEEE Trans. on Auto. Contr., Vol. AC-24, pp. 926-932, December 1979.

B. Additional Reports and Conference Proceedings

Pearson, A. E. "Identification of Linear Differential Systems from Input-Output Data Without Estimating the Initial State," Proc. of 1975 Conf. on Info. Sci. and Syst., pp. 387-390, Johns Hopkins U., April 1975.

Pearson, A. E. "Positive Definite Performance Functions for Parameter Adaptive Control Problems," Proc. of 1975 IEEE Conf. on Decis. and Contr., pp. 844-849, Houston, Texas, December 1975.

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Chin, Y. K. and Pearson, A. E., "Computational Aspects of Finite Time Interval Identification Without Initial State Estimation," Proc. of 1977 IEEE Conf. on Decis. and Contr., New Orleans, Louisiana, pp. 892-897, December 1977.

C. M.S. and Ph.D. Theses

Koopersmith, R. M., "Determination of Trajectories for a Gliding Parachute System," M.S. Thesis, June, 1975.

Kwon, W. H., "Infinite Time Regulator for a Class of Functional Differential Systems and a Minimum Control Energy Problem for Ordinary Differential Systems," Ph.D. Thesis, June 1976.

Wei, K. C., "Optimal Control of Bilinear Systems With Some Aerospace Applications," Ph.D. Thesis, June 1976.

Chin, Y. K., "Theoretical and Computational Considerations for Finite Time Interval System Identification Without State Estimation," Ph.D. Thesis, June 1977.

Mocenigo, J. M. "A Deterministic Filter from a Parameter Identification Scheme," M.S. Thesis, June 1978.

Yu, Kai-bor, "Modeling, Estimation and Control of a Two Dimensional Intercept System," M.S. Thesis, June 1979.

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